DISSERTATIO MATHEMATICA:

RATIONEM, DIFFERENTIAM FUNCTIONUM, E DIFFERENTIALIBUS EARUMDEM DETERMINANDI, OSTENDENS,

QUAM

CONSENSU AMPLISSIMÆ FACULTATIS PHILOSOPHICÆ,

PRÆSIDE

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ABOÆ, typis *Frenckellianis*.

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Relationem inter differentias et differentialia functionum investigare, respectu usus, quem expressiones generales pro differentiis inventæ habent, momenti magni esse videtur. Quæ relatio, ipsa ratione, qua ope differentialium, formula differentias functionum definiens obtinetur, accuratius considerata, facilius perspici potest. Formula ea viis quidem duabus resultat, licet altera principiis generalibus cum careat, methodus demonstrandi appellari non possit; unde formulam, quæ ea reportatur via, generalem considerare non liceret, nisi altera, eandem generaliori modo demonstraret. Quarum illam, quæ theoremate binomiali Newtoniano nititur, prius ope differentiarum ostendentes regulas binomiales esse universales, primum afferamus. Regulæ vero binomiales modo sequenti explicari possunt. Sit x quantitas variabilis, supponamusque differentiam terminorum x, $x + \Delta x$, $x + 2\Delta x$, $x + 3\Delta x$, $x + 4\Delta x$, $x + 5\Delta x$, $x + 6\Delta x$, qui in serie arithmetica sequuntur, per Δx insignitam, quantitatem esse finitam: functiones vero, quæ quantitatibus x, $x + \Delta x$, $x + 2\Delta x$, $x + 3\Delta x$, $x + 4\Delta x$, $x + 5\Delta x$, $x + 6\Delta x$, ad potentiam n elevatis respondent, brevitatis ergo per y, y^{I}, y^{II}, y^{III}, y^{IV}, y^{V}, y^{VI} insigniamus. Indicent ulterius Δy, Δy^{I}, Δy^{II}, Δy^{III}, Δy^{IV}, Δy^{V} differentias, quæ inter radices x, $x + \Delta x$, $x + 2\Delta x$ &c., elevatas ad po-

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tentiam n, h. e. inter x^n, $(x + \Delta x)^n$, $(x + 2\Delta x)^n$, $(x + 3\Delta x)^n$, $(x + 4\Delta x)^n$, $(x + 5\Delta x)^n$, $(x + 6\Delta x)^n$ reperiuntur: vel, quod eodem redit, Δy, Δy^I, Δy^{II} &c. significent quantitates ante functionibus y, y^I, y^{II} &c. addendas, quam functiones y^I, y^{II}, y^{III} &c. resultant. Eodem modo designent $\Delta^2 y$, $\Delta^2 y^I$, $\Delta^2 y^{II}$, $\Delta^2 y^{III}$, $\Delta^2 y^{IV}$, $\Delta^2 y^V$ quantitates, quas prius differentiis Δy, Δy^I, Δy^{II}, Δy^{III}, Δy^{IV}, Δy^V, addere debemus, quam ex datis $\Delta^2 y$ et Δy, Δy^I; ex $\Delta^2 y^I$ et Δy^I, Δy^{II}; ex $\Delta^2 y^{II}$ et Δy^{II}, Δ^{III} &c. obtinere possumus. Hic differentias determinandi modus permitti etsi possit ostendenti valorem alicujus functionum y^I, y^{II}, y^{III}, y^{IV}, y^V, y^{VI}, ope differentiarum determinatum, æqualem esse valori, qui ope theorematis binomialis Newtoniani definitus, eidem respondet; differentiæ tamen eædem, si Δy^I, datis Δy et $\Delta^2 y$, vel $\Delta^2 y$, datis Δy^I et Δy determinantur, cum fiant, vocabulumque *differentia* in *calculo differentiarum* in primis de quantitatibus per *subtractionem* ortis usurpetur; differentias Δy, Δy^I, Δy^{II}, Δy^{III}, Δy^{IV}, Δy^V, quantitates ortas per subtractionem uniuscujusque t rmini antecedentis e subsequenti in serie y, y^I, y^{II}, ^{III}, y^{IV}, y^V, y^{VI}, indicare, dicere licet. Pari ratione $\Delta^2 y$, $\Delta^2 y^I$, $\Delta^2 y^{II}$, $\Delta^2 y^{III}$, $\Delta^2 y^{IV}$ insigniunt quantitates ortas per subtractionem uniuscujusque termini antecedentis e subsequenti in serie Δy, Δy^I, Δy^{II}, Δy^{III}, Δy^{IV}, Δy^V. Hocde $\Delta^3 y$, $\Delta^3 y^I$, $\Delta^3 y^{II}$, $\Delta^3 y^{III}$; $\Delta^4 y$, $\Delta^4 y^I$, $\Delta^4 y^{II}$; $\Delta^5 y$, $\Delta^5 y^I$; $\Delta^6 y$ quoque valet. Ope harum differentiarum, unaquæque functionum y^I, y^{II}, y^{III}, y^{IV}, y^V, y^{VI} determinari potest. Exempli gratia $y^{VI} = y^V + \Delta^6 y + 5\Delta^5 y + 10\Delta^4 y + 10\Delta^3 y + 5\Delta^2 y + \Delta y$ reportatur. Est nempe $y^{VI} = y^V + \Delta y^V$; $\Delta y^V = \Delta^2 y^{IV} + \Delta y^{IV} = \Delta^3 y^{III} + 2\Delta^2 y^{III} + \Delta y^{III} = \Delta^4 y^{II} + 3\Delta^3 y^{II} + 3\Delta^2 y^{II} + \Delta y^{II} = \Delta^5 y^I + 4\Delta^4 y^I + 6\Delta^3 y^I + 4\Delta^2 y^I + \Delta y^I = \Delta^6 y + 5\Delta^5 y + 10\Delta^4 y + 10\Delta^3 y + 5\Delta^2 y + \Delta y$. Quo postremo valore pro Δy^V substituto, loco y^{VI} valor allatus resultat. Valoribusque, quos differentiæ $\Delta^6 y$, $\Delta^5 y$, $\Delta^4 y$, $\Delta^3 y$, $\Delta^2 y$, Δy posito n ex. gr $= 6$ obtinent, secundum formulam differentiarum generalem

lem $\Delta^m y = Pa x^{n-m} \Delta x^m + Q\beta x^{n-m-1} \Delta x^{m+1} + R\gamma x^{n-m-2} \Delta x^{m+2}$ &c. determinatis; $\Delta y = 6x^5 \Delta x + 15x^4 \Delta x^2 + 20x^3 \Delta x^3 + 15x^2 \Delta x^4 + 6x \Delta x^5 + \Delta x^6$; $5\Delta^2 y = 150x^4 \Delta x^2 + 600x^3 \Delta x^3 + 1050x^2 \Delta x^4 + 900x \Delta x^5 + 310 \Delta x^6$; $10\Delta^3 y = 1200x^3 \Delta x^3 + 5400x^2 \Delta x^4 + 9000x \Delta x^5 + 5400 \Delta x^6$; $10\Delta^4 y = 3600x^2 \Delta x^4 + 14400x \Delta x^5 + 15600 \Delta x^6$; $5\Delta^5 y = 3600x \Delta x^5 + 9000 \Delta x^6$; $\Delta^6 y = 720 \Delta x^6$ obtinemus. Regulæ vero binomiales nulla ſub conditione univerſales esſe posſunt, niſi valor qui ſecundum easdem pro $y^{VI} = (x + 6\Delta x)^6 = ((x + 5\Delta x) + \Delta x)^6$ reportatur, æqualis foret valori ope differentiarum invento. Secundum theorema binomiale reperiuntur: $(x + 5\Delta x) + \Delta x)^6 = (x + 5\Delta x)^6 + 6(x + 5\Delta x)^5 \Delta x + 15(x + 5\Delta x)^4 \Delta x^2 + 20(x + 3\Delta x)^3 \Delta x^3 + 15(x + 5\Delta x^2 \Delta x^4 + 6(x + 5\Delta x) \Delta x^5 + \Delta x^6$; $6(x + 5\Delta x)^5 \Delta x = 6x^5 \Delta x + 6.5.5x^4 \Delta x^2 + 6.10.5^2 x^3 \Delta x^3 + 6.10.5^3.x^2 \Delta x^4 + 6..5.5^4.x \Delta x^5 + 6.5^5 \Delta x^6$; $15(x + 5\Delta x)^4 \Delta x^2 = 15x^4 \Delta x^2 + 15.4.5.x^3 \Delta x^3 + 15.6.5^2 x^2 \Delta x^4 + 15.4.5^3 x \Delta x^5 + 15.5^4 \Delta x^6$; $20(x + 5\Delta x)^3 \Delta x^3 = 20x^3 \Delta x^3 + 20.3.5 x^2 \Delta x^4 + 20.3.5^2 x \Delta x^5 + 20.5^3 \Delta x^6$; $15(x + 5\Delta x)^2 \Delta x^4 = 15x^2 \Delta x^4 + 15.2.5x \Delta x^5 + 15.5^2 \Delta x^6$; $6(x + 5\Delta x) \Delta x^5 = 6x \Delta x^5 + 6.5 \Delta x^6$. Posſumus vero loco y^V in æquatione $y^{VI} = y^V + \Delta^6 y + 5\Delta^5 y + 10\Delta^4 y + 10\Delta^3 y + 5\Delta^2 y + \Delta y$; $(x + 5\Delta x)^6$ ponere. Supra etenim $y^V = (x + 5\Delta x)^n$ poſuimus, unde poſito $n = 6$, $y^V = (x + 5\Delta x)^6$ oritur. Reliqui vero valores functioni y^{VI} reſpondentes ope differentiarum et theorematis binomialis *Newtoniani* enumerati æquales esse debent. Hunc, ope differentiarum, functionum y^I, y^{II}, y^{III}, y^{IV}, y^V, y^{VI} quandam determinandi, modum, propter ipſas differentias, quæ ſine calculo differentiali difficilius determinantur, minus aptum esſe, ex antecedentibus patet. Expoſita vero, ope Calculi differentialis, formula generali, ſecundum quam, differentia finita functionis y differentiæ finitæ ipſius x vel Δx reſpondens inveniri poteſt, unaquæque functionum y^I, y^{II}, y^{III}, y^{IV}, y^V, y^{VI} &c. facillime determinatur. Hujus formulæ obtinendæ gratia,

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ponamus $y = x^n$; $y^I = (x + \Delta x)^n$. E posteriori æquatione, ope theorematis binominalis *Newtoniani*, obtinetur $y^I = x^n$

$$+ \frac{n}{1} x^{n-1}\Delta x + \frac{n}{1}\left(\frac{n-1}{2}\right) x^{n-2}\Delta x^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} x^{n-3}\Delta x^3$$

$$+ \frac{n(n-1)(n-2)\,n-3)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot} x^{n-4}\Delta x^4 + \frac{n(n-1)(n-2)(n-3)(n-4)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot} x^{n-5}\Delta x^5$$

&c. Unde $(x + \Delta x)^n - x^n = y^I - y = \Delta y = \frac{n}{1} x^{n-1}\Delta x$

$$+ \frac{n}{1} \text{[illegible]}\,{}^{2}\Delta x^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3 \cdot} x^{n-3}\Delta x^3$$

$$+ \frac{n(n - \text{[illegible]}}{1 \quad 2 \quad \quad 4} x^{n-4}\Delta x^4 + \frac{n(n-1)(n-2)(n-3)(n-4)}{1 \quad 2 \quad 3 \quad 4 \quad 5} x^{n-5}\Delta x^5$$

&c. Differentialia vero functionis $y = x^n$, posito dx constanti, fiunt:

$$\frac{dy}{dx} = nx^{n-1};\ \frac{d^2y}{dx^2} = n(n-1)\,x^{n-2};\ \frac{d^3y}{dx^3} = n(n-1)(n-2)\,x^{n-3};$$

$$\frac{d^4y}{dx^4} = n(n-1)(n-2)(n-3)\,x^{n-4};\ \frac{d^5y}{dx^5} = n(n-1)(n-2)(n-3)(n-4)x^{n-5}.$$

Quibus quantitatibus in formula $\Delta y = \frac{n}{1} x^{n-1}\Delta x + \frac{n(n-1)}{1 \quad 2} x^{n-2}\Delta x^2$

$$+ \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} x^{n-3}\Delta x^3 + \frac{n(n-1)(n-2)(n-3)}{1 \cdot 2 \cdot 3 \cdot 4} x^{n-4}\Delta x^4$$

$+ \frac{n(n-u)(n-2)(n-3)(n-4)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} x^{n-5}\Delta x^5$ &c. adhibitis: Δy

$$= \frac{dy\Delta x}{1 \, . \, ax} + \frac{d^2y\Delta x^2}{1.2.dx^2} + \frac{d^3y\Delta x^3}{1.2.3dx^3} + \frac{d^4y\Delta x^4}{1.2.3.4dx^4} + \frac{d^5y\Delta x^5}{1.2.3.4.5dx^5} \text{ \&c.}$$

reportatur. Hanc formulam longe utilissiman, cujus usus, series summatrices, maximos atque minimos valores quantitatum variabilium determinandos, radicesque æquationum superiorum graduum per approximationem inveniendas, aliasque res in calculo differentiali occurrentes si respexeris, haud minimus erit; generaliori modo demonstratam in libris calculum differentialem tractantibus reperi. Methothodus, qua ab EULERO *a*) ſine ope theorematis binomialis *Newtoniani*, formula ſupra allata demonſtratur, generalior demonſtratione a nobis allata eſt. Ipſum tamen nervum methodi, qua EULERUS formulam, quæ differentiam finitam functionis exprimit, demonſtrat, cum non ſatis perſpiciamus; demonſtrationem, quam EULERUS forſan cogitavit, proferre audebimus. Hunc in finem, $x^3 + mx$, functionem ipſius x ponamus. Hanc functionem cum conſideramus, [illegible] via, qua demonſtrari posſent ea, quæ EULERUS d[illegible] voluit, ſequens nobis viſa eſt. Cog[illegible]verſum quantitatem variabilem x, differentialeque [illegible], necesſario ex illa relatione, quæ inter x et function[illegible] $x^3 + mx$ intercedit, notioneque cum vocabulo differentialis conjungenda, ſequitur, ut differentiale functionis $x^3 + mx$, $(3x^2 + m)$ dx fiat. Quod hoc differentiale inſimul relationem inter differentiale ipſius x et differentiale functionis $x^3 + mx$ indicet, facile videtur. Indicatque $(3x^2 + m)\, dx$ differentiale, quod, ſi quantitati x tribuerentur valores, quorum differentiale dx esſet, inter valores functionis $x^3 + mx$ exiſteret. Determinantes itaque valorem functionis $x^3 + mx$ e quodam valore $= x$, valorem functionis $x^3 + mx + (3x^2 + m)\, dx$ ex eodem valore $+\, dx$ vel ex $(x + dx) = x'$ determinare debemus. Ponamus iterum differentiale, quod intercedit inter $(x + dx) = x'$, et valorem proximo majorem, quem per x'' inſignimus, esſe dx vel $x'' - x' = dx$, conſideremusque functionem $x^3 + mx + (3x^2 + m)\, dx$ reſpectu differentialis, quod cogitamus esſe inter x' et x'' differentiale functionis $x^3 + mx + (3x^2 + m)\, dx$, poſito dx conſtanti, $(3x^2 + m)$ $dx + 6dx^2$ fieri, videre licet. In calculo etenim differentiali quæſtio omnino non eſt, quem valorem quantitas x indicet; quæritur modo differentiale functionis cujuscunque ipſius x,

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a) V. Calcul. differ. p. 332 ſqq.

posito differentiam, per dx insignitam, duorum valorum, qui quantitati variabili x tribuerentur, esse infinite parvam. Igitur dx indicat differentiam infinite parvam, quam cogitamus interesse inter primam dignitatem quantitatum duarum quarumcunque, quae loco x poni possent. Talem vero differentiam infinite parvam, quae inter primam dignitatem duarum quantitatum quarumcunque, quas per x et $x + dx$ insignire volumus, cogitare, nullius foret frugis, nisi ex hac notione differentiae infinite parvae vel differentialis inter x et $x + dx$ intercedentis, differentiale inter x^n et $(x + dx)^n$; inter $x^{\frac{1}{n}}$ et $(x + dx)^{\frac{1}{n}}$ &c. intercedens multo brevius, quam sine hac notione differentiae infinite parvae fieri posset, exprimere possemus. Itaque nulla contrarietas in ea re est, quod differentiale functionis $x^3 + mx + (3x^2 + m)\, dx$, $(3x^2 + m)\, dx + 6xdx^2$, posito dx constanti, fieri statuamus. Unde ex functione $x^3 + mx + (3x^2 + m)\, dx$, posito loco x' x'', functionem $x^3 + mx + 2 \,.\, (3v^2 + m)\, dx + 6xdx^2$, reportamus. Ponamus iterum differentiale inter x'' et valorem, proximo majorem, quem per x''' insignimus, esse dx vel $x''' - x'' = dx$. Additisque functione $x^3 + mx + 2 \,.\, (3x^2 + m)\, dx + 6xdx^2$ et differentiali ipsius functionis: $x^3 + mx + 3\, (3x^2 + m)\, dx + 3 \,.\, 6xdx^2 + 6dx^3$ obtinentur. Quod haec functio, posito x''' loco x'' reportetur, e supra expositis patet.

Si denuo ponamus differentiale inter x''' et valorem proximo majorem, quem iterum per x^{IV} insignimus, esse dx vel $x^{IV} - x^{III} = dx$, valorem functionis, qui respondet valori x^{IV}, loco x^{III} posito, $x^3 + mx + 4 \,.\, (3x^2 + m)\, dx + 6 \,.\, 6xdx^2 + 4 \,.\, 6dx^3$ obtinemus. Pari ratione posito $x^V - x^{IV} = dx$, valor functionis respondens valori per x^V insignito, foret $x^3 + mx + 5\, (3x^2 + m)\, dx + 10 \,.\, 6xdx^2 + 10 \,.\, 6dx^3$.

Confiderantes has functiones, valoribus per x, x^{I}, x^{II}, x^{III} &c. infignitis, refpondentes; differentialia functionum duobus proximis valoribus, quorum differentiale idem eft respondentium, æqualia non esfe videmus. Hacque re demonftratio ex parte nititur. Causfam vero cur in exemplo fupra allato differentiale intercedens inter functiones valoribus x^{V} et x^{IV} refpondentes, majus fit differentia infinite parva inter functiones valoribus x^{IV} et x^{III} refpondentes intercedente, hæcque differentia iterum major differentia, quæ intercedit inter functiones, valoribus x^{III} et x^{II} refpondentes; functionem originalem $x^3 + mx$ confiderando, facile perfpicimus. Sunt enim exponentes ambarum quantitatum x^3 et x numeri pofitivi et toti. Facile namque videre licet differentiam duarum quantitatum ex. gr. x^{V} et x^{IV} elatarum ad altiores potentias pofitivas majorem esfe differentia, quæ intercedit inter duas quantitates minores, ex. gr. x^{IV} et x^{III} elatas ad easdem potentias pofitivas, etfi differentia intercedens inter x^{V} atque x^{IV}, interque x^{IV} et x^{III}, eadem esfe posfit. Coëfficientes vero terminorum occurrentium in functionibus, quæ valoribus x^{II}, x^{III}, x^{IV}, x^{V} respondent, contemplantes, feriem eandem, quam coëfficientes terminorum binomii elevati ad dignitates 2, 3, 4, 5 obfervant, reperimus. Eodemque, quo in præcedentibus, progredientes modo, ponentesque differentiale inter x^{VI} et x^{V}; inter x^{VII} et x^{VI}; inter x^{VIII} et x^{VII} &c. $= dx$: valorem functionis refpondentis ex. gr. valori x^{VIII}, confiderando feriem coëfficientium terminorum in functionibus fupra allatis: $x^3 + mx + \frac{8}{1}(3x^2 + m)\,dx + \frac{8 \cdot 7}{1 \cdot 2}\,6x\,dx^2 + \frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3}\,6dx^3$, fore videmus. Si vero in functione originali $x^3 + mx$, loco quantitatis x^3, ex. gr. x^4 fuisfet, in functione, quæ tunc valori x^{VIII} refponderet, dx^4 infimul occurreret, coëfficiensque termini, in quo dx^4 occurreret, $\frac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4}$

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foret. Nam facile videre posſumus, quod, poſito x^4 loco x^3 in exemplo pluries allato, loco quantitatum $(3x^2 + m)\,dx$, $6xdx^2$, $6dx^3$ quantitates $(4x^3 + m)\,dx$, $12x^2dx^2$, $24xdx^3$ ſcribendæ forent; modo conſideremus, quod, per $(3x^2+m)\,dx$, differentiale functionis originalis; per $6xdx^2$, differentiale functionis per primam differentiationem ortæ; per $6dx^3$, differentiale functionis per ſecundam differentiationem ortæ, generaliter indicentur. In quo reſpectu, poſito x^4 loco x^3 in functione $x^3 + mx$; $24dx^4$ generaliter indicaret differentiale functionis per tertiam differentiationem ortæ. Unde, inſignita tota functione per y; has quantitates e continua differentiatione functionis originalis dependentes per dy, dy^2, dy^3, dy^4 &c. apte indicare posſumus. Ex ſupra allatis inſimul perſpicimus, quod, poſito omnes exponentes quantitatis x in functione originali esſe numeros poſitivos et totos, et dx conſtans, exponentemque maximum $= n$; differentiale inſigniendum per $d^{n+1}y$ omnino evaneſcat. Posſumus itaque, totam functionem originalem per y indicantes, functionem valori x^{VIII} reſpondentem; per $y + \frac{8}{1}\,dy + \frac{8 \cdot 7}{1 \cdot 2}\,d^2y + \frac{8 \cdot 7 \cdot 6}{1 \cdot 2 \cdot 3}\,d^3y + \frac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4}\,d^4y$ &c. inſignire. E ſerie vero valorum per x, x^I, x^{II}, x^{III} . . . x^{VIII} inſignitorum, quorum differentia $= dx$ eſt; differentiale intercedens inter x et $x^{II} = 2dx$; inter x et $x^{III} = 3dx$; inter x et $x^{VIII} = 8dx$, esſe videmus. Unde functio allata valori $x + 8dx$ reſpondet. In ſerie valorum, qui loco x poni posſunt continue progredientes ſemperque differentiale inter duos proximos valores $= dx$ ponentes, ad valorem inter quem et valorem relative per x inſignitum, differentia, quam generaliter per Δx inſignire posſumus interesſet, perveniremus. Qui ergo valor per $x + \Delta x$ indicandus eſt. Numerum vero terminorum talis ſeriei arithmeticæ, in qua differentia terminorum $= dx$ eſt, modo differentiam inter terminos per x et $x + \Delta x$ inſignitos finitam ponamus, infinite magnum conſiderare debemus. Indicatoque numero terminorum, quem ſupponimus infinite magnum per n; terminus

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insignitus per $x + \Delta x$ foret $= x + ndx$ Considerantesque functionem valori $x + 8dx$ respondentem; functionem, quæ valori $x + ndx$ loco x posito responderet; $y + \frac{n}{1} dy + \frac{n(n-1)}{1.2} d^2y + \frac{n(n-1)(n-2)}{1.2.3} d^3y + \frac{n(n-1)(n-2)(n-3)}{1.2.3.4} d^4y$ &c. futuram, videmus. Ponentes vero $ndx = \Delta x$, quantitatem n, ut jam supra diximus, infinite magnam considerare possumus. In quo respectu loco quantitatum $n - 1$, $n - 2$, $n - 3$ &c. quantitatem n ponere licet. Ex æquatione $ndx = \Delta x$ reportantur $n = \frac{\Delta x}{dx}$; $n^2 = \frac{\Delta x^2}{dx^2}$; $n^3 = \frac{\Delta x^3}{dx^3}$; $n^4 = \frac{\Delta x^4}{dx^4}$ &c. Quatenus itaque $x + \Delta x$ pro $x + ndx$ ponitur; loco functionis $y + \frac{n}{1} dy + \frac{n(n-1)}{1.2} d^2y + \frac{n(n-1)(n-2)}{1.2.3} d^3y + \frac{n(n-1)(n-2)(n-3)}{1.2.3.4} d^4y$ &c. formula $y + \frac{\Delta x dy}{1.dx} + \frac{\Delta x^2 d^2y}{1.2.dx^2} + \frac{\Delta x^3 d^3y}{1.2.3dx^3} + \frac{\Delta x^4 d^4y}{1.2.3.4dx^4}$ &c., obtinetur. Unde $\Delta y = \frac{\Delta x dy}{1.dx} + \frac{\Delta x^2 d^2y}{1.2.dx^2} + \frac{\Delta x^3 d^3y}{1.2.3dx^3} + \frac{\Delta x^4 d^4y}{1.21.3.4.dx^4}$ &c.

Hujus formulæ illustrandæ gratia, $y = x^6 + mx^4 + rx$, ponamus. Differentia hujus functionis secundum formulam, $\Delta y = \frac{\Delta x dy}{1.dx} + \frac{\Delta x^2 d^2y}{1.2.dx^2} + \frac{\Delta x^3 d^3y}{1.2.3dx^3} + \frac{\Delta x^4 d^4y}{1.2.3.4dx^4} + \frac{\Delta x^5 d^5y}{1.2.3.4.5dx^5}$ &c. determinata, foret; $\frac{\Delta x}{1} (6x^5 + 4mx^3 + r) + \frac{\Delta x^2}{1.2} (6.5.x^4$

 + 12.

$+ 12mx^2) + \frac{\Delta x^3}{1.2.3}(6.5.4x^3 + 2.12mx) + \frac{\Delta x^4}{1.2.3.4}(6.5.4.3x^2$ $+ 2.12m) + \frac{\Delta x^5}{1.2.3.4.5}.6.5.4.3.2x + \Delta x^6$. In hoc enin casu $\frac{dy}{dx} = 6x^5 + 4mx^3 + r$; $\frac{d^2y}{dx^2} = 6.5x^4 + 12mx^2$; $\frac{d^3y}{dx^3}$ $= 6.5.4x^3 + 2.12mx$; $\frac{d^4y}{dx^4} = 6.5.4.3x^2 + 2.12m$; $\frac{d^5y}{dx^5}$ $= 6.5.4.3.2x$; $\frac{d^6y}{dx^6} = 6.5.4.3.2.1$; $\frac{d^7y}{dx^7} = 0$, funt. Obfervandum enim eft, quod $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, $\frac{d^3y}{dx^3}$, $\frac{d^4y}{dx^4}$, $\frac{d^5y}{dx^5}$, $\frac{d^6y}{dx^6}$ &c., e functione, cujus differentiam quærimus, determinare debeamus. Hæc differentia, pofito $x + \Delta x$ loco x, fecundum theorema binomiale Newtonianum definita, esfet; $6x^5\Delta x + 15x^4\Delta x^2$ $+ 20x^3\Delta x^3 + 15x^2\Delta x^4 + 6x\Delta x^5 + \Delta x6 + 4mx^3\Delta x$ $+ 6mx^2\Delta x^2 + 4mx\Delta x^3 + m\Delta x^4 + \Delta xr$. Quod hæc differentia fecundum formulam allatam et theorema binomiale Newtonianum determinata, æqualis fit, facile apparet. Unde expresfio $\Delta y = \frac{\Delta x dy}{1.dx} + \frac{\Delta x^2 d^2y}{1.2dx^2} + \frac{\Delta x^3 d^3y}{1.2.3dx^3}$ $+ \frac{\Delta x^4 d^4y}{1.2.3.4dx^4} + \frac{\Delta x^5 d^5y}{1.2.3.4.5dx^5}$ &c. hoc modo probari poteft.

Ope hujus formulæ functiones, quas fupra *b*) per y^I, y^{II}, y^{III}, y^{IV}, y^V, y^{VI} infignivimus, modo fequenti determinantur:

$$y^I = y + \frac{\Delta x dy}{1.dx} + \frac{\Delta x^2 d^2y}{1.2dx^2} + \frac{\Delta x^3 d^3y}{1.2.3dx^3} + \frac{\Delta x^4 d^4y}{1.2.3.4dx^4}$$

b) Vide paginam primam.

$$+ \frac{\Delta x^5 d^5 y}{1.2.3.4.5 dx^5} \text{ \&c. } y^{II} = y + \frac{2\Delta x dy}{1 . dx} + \frac{4\Delta x^2 d^2 y}{1 . 2dx^2}$$

$$+ \frac{8\Delta x^3 d^3 y}{1.2.3dx^3} + \frac{16\Delta x^4 d^4 y}{1.2.3.4dx^4} + \frac{32\Delta x^5 d^5 x}{1.2.3.4.5dx^5} \text{ \&c. } y^{III} = y$$

$$+ \frac{3\Delta x dy}{1 . dx} + \frac{9\Delta x^2 d^2 y}{1 . 2dx^2} + \frac{27\Delta x^3 d^3 y}{1.2.3dx} + \frac{81\Delta x^4 d^4 y}{1.2.3.4dx^4}$$

$$+ \frac{243\Delta x^5 d^5 y}{1.2.3.4.5dx^5} \text{ \&c. } y^{IV} = y + \frac{4\Delta x dy}{1 . dx} + \frac{16\Delta x^2 d^2 y}{1 . 2dx^2}$$

$$+ \frac{64\Delta x^3 d^3 y}{1.2.3dx^3} + \frac{256\Delta x^4 d^4 y}{1.2.3.4dx^4} \text{ \&c. } y^{V} = y + \frac{5\Delta x dy}{1 . 2dx}$$

$$+ \frac{25\Delta x^2 d^2 y}{1 . 2dx^2} + \frac{125\Delta x^3 d^3 y}{1.2.3dx^3} + \frac{625\Delta x^4 d^4 y}{1.2.3.4dx^4} \text{ \&c. } y^{VI} = y$$

$$+ \frac{6\Delta x dy}{1 . dx} + \frac{36\Delta x^2 d^2 y}{1 . 2dx^2} + \frac{216\Delta x^3 d^3 y}{1.2.3dx^3} + \frac{1296\Delta x^4 d^4 y}{1.2.3.4dx^4} \text{ \&c.}$$

Quod expresſiones allatæ e functione, $y = x^n$, poſito $x + \Delta x$, $x + 2\Delta x$, $x + 3\Delta x$, $x + 4\Delta x$, $x + 5\Delta x$, $x + 6\Delta x$ loco x, oriantur, patet.

Corrigenda:

Pag. 2 lin. 9 Δ''' leg. $\Delta y'''$
— 3 — 11 $(x + 5\Delta x) + \Delta x)^6$ leg. $((x + 5\Delta x) + \Delta x)^6$
— 5 — 26 $(3x^2 + m, dx$ leg. $(3x^2 + m)\, dx$
— — — 28 $(3x^2 + m\, dx$ leg. $(3x^2 + m)\, dx$
— 6 — 17 $(3v^2 + m)\, dx$ leg. $(3x^2 + m)\, dx$
— 8 — 1. Nam facile leg. Facile ſimul
— — — 12 dy^2, dy^3, dy^4 leg. d^2y, d^3y, d^4y.